Topic: Normal lines

Question: Find the equation of the normal line to the function at the given point.

$$y = 2x^4$$

at $(1,2)$

Answer choices:

A $y = 8x - 6$

B $y = -\frac{1}{8}x - \frac{17}{8}$

C $y = -\frac{1}{8}x + \frac{17}{8}$

D $y = 8x - 10$

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Solution: C

The normal line is the line that's perpendicular to the tangent line. If we can find the tangent line to a function at a particular point, then we can also find the normal line.

To find the equation of a normal line, we need to first find the derivative of our function. This will give us the slope of the tangent line at the given point. Then, we take the negative reciprocal of the slope of the tangent line to get the slope of the normal line. Finally, we can use the given point, the slope of the normal line, and the point-slope formula of the equation of a line to generate the normal line equation.

Let's start by using power rule to take the derivative of our function $y = 2x^4$.

$$y' = 8x^3$$

Plugging the given point $(1,2)$ into our derivative, we'll get the slope of the tangent line, m.

$$m = 8(1)^3$$

$$m = 8$$

We'll take the negative reciprocal of the slope of the tangent line to get the slope of the normal line.

$$m_n = -\frac{1}{m}$$

$$m_n = -\frac{1}{8}$$

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Finally, we can use the point $(1,2)$, the slope of the normal line $m_n = -1/8$, and the point-slope formula $(y - y_1) = m(x - x_1)$ to generate the equation of the normal line.

$$(y - 2) = -\frac{1}{8}(x - 1)$$

$$y = -\frac{1}{8}x + \frac{1}{8} + 2$$

$$y = -\frac{1}{8}x + \frac{17}{8}$$

The equation of the normal line to the function $y = 2x^4$ at the point $(1,2)$ is

$$y = -\frac{1}{8}x + \frac{17}{8}$$

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Topic: Normal lines

Question: Find the equation of the normal line to the function at the given point.

$$y = x\sqrt{x+1}$$

at $(3,6)$

Answer choices:

A $y = -\frac{4}{11}x + \frac{78}{11}$

B $y = \frac{11}{4}x - \frac{57}{4}$

C $y = \frac{11}{4}x - \frac{9}{4}$

D $y = -\frac{4}{11}x - \frac{54}{11}$

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Solution: A

The normal line is the line that's perpendicular to the tangent line. If we can find the tangent line to a function at a particular point, then we can also find the normal line.

To find the equation of a normal line, we need to first find the derivative of our function. This will give us the slope of the tangent line at the given point. Then, we take the negative reciprocal of the slope of the tangent line to get the slope of the normal line. Finally, we can use the given point, the slope of the normal line, and the point-slope formula of the equation of a line to generate the normal line equation.

Let's start by using product rule to take the derivative of our function $y = x\sqrt{x+1}$.

$$y' = (1)\left(\sqrt{x+1}\right) + (x)\left[\frac{1}{2}(x+1)^{-\frac{1}{2}}(1)\right]$$

$$y' = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$$

$$y' = \frac{2(x+1)+x}{2\sqrt{x+1}}$$

$$y' = \frac{3x+2}{2\sqrt{x+1}}$$

Plugging the given point $(3,6)$ into our derivative, we'll get the slope of the tangent line, m.

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$$m = \frac{3(3)+2}{2\sqrt{3+1}}$$

$$m = \frac{11}{4}$$

We'll take the negative reciprocal of the slope of the tangent line to get the slope of the normal line.

$$m_n = -\frac{1}{m}$$

$$m_n = -\frac{4}{11}$$

Finally, we can use the point $(3,6)$, the slope of the normal line $m_n = -4/11$, and the point-slope formula $(y - y_1) = m(x - x_1)$ to generate the equation of the normal line.

$$(y-6) = -\frac{4}{11}(x-3)$$

$$y = -\frac{4}{11}x + \frac{12}{11} + 6$$

$$y = -\frac{4}{11}x + \frac{78}{11}$$

The equation of the normal line to the function $y = x\sqrt{x+1}$ at the point $(3,6)$ is

$$y = -\frac{4}{11}x + \frac{78}{11}$$

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Topic: Normal lines

Question: Find the equation of the normal line to the function at the given point.

$$y = \frac{2x^2}{x+2}$$

at $(2,2)$

Answer choices:

A $y = -\frac{2}{3}x - \frac{2}{3}$

B $y = \frac{3}{2}x - 1$

C $y = \frac{3}{2}x - 5$

D $y = -\frac{2}{3}x + \frac{10}{3}$

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Solution: D

The normal line is the line that's perpendicular to the tangent line. If we can find the tangent line to a function at a particular point, then we can also find the normal line.

To find the equation of a normal line, we need to first find the derivative of our function. This will give us the slope of the tangent line at the given point. Then, we take the negative reciprocal of the slope of the tangent line to get the slope of the normal line. Finally, we can use the given point, the slope of the normal line, and the point-slope formula of the equation of a line to generate the normal line equation.

Let's start by using quotient rule to take the derivative of the function

$$y = \frac{2x^2}{x+2}$$

$$y' = \frac{(4x)(x+2) - \left(2x^2\right)(1)}{(x+2)^2}$$

$$y' = \frac{4x^2 + 8x - 2x^2}{(x+2)^2}$$

$$y' = \frac{2x^2 + 8x}{(x+2)^2}$$

Plugging the given point $(2,2)$ into the derivative, we'll get the slope of the tangent line, m.

$$m = \frac{2(2)^2 + 8(2)}{(2+2)^2}$$

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$$m = \frac{24}{16}$$

$$m = \frac{3}{2}$$

We'll take the negative reciprocal of the slope of the tangent line to get the slope of the normal line.

$$m_n = -\frac{1}{m}$$

$$m_n = -\frac{2}{3}$$

Finally, we can use the point $(2,2)$, the slope of the normal line $m_n = -2/3$, and the point-slope formula $(y - y_1) = m(x - x_1)$ to generate the equation of the normal line.

$$(y - 2) = -\frac{2}{3}(x - 2)$$

$$y = -\frac{2}{3}x + \frac{4}{3} + 2$$

$$y = -\frac{2}{3}x + \frac{10}{3}$$

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